

CRYPTOGRAPHY

(lecture 8)

Literature:

“To Signal or Not to Signal?..” By Nelson, Pagnin, Askarov (**Section 3**)

“Analysing the Signal Protocol”, D. Van Dan (ch 2.1.0-2.1.3)

“Post Quantum Cryptography”, by T. Frederiksen

“A Graduate Course in Applied Cryptography” (ch **14.0,14.1.0,8.3**)

Announcements

- **Office Hours! Ivan** from next week (announcement on Canvas later today)
- Lecture on Dec 13th: course **recap** + info about **exam**
- Lecture on Dec 9th: by **Victor** on ABC
- All next exercise sessions by **William**
- HA2 deadline: Tuesday **29th** (1st submission)
- Bonus 2 deadline: Friday **2nd**
- HA3 deadline: Tuesday **13th** (1st submission)
- Bonus 3 deadline: Tuesday **20th**
- HARD deadline for final submission: **Jan 8th** (Sunday)
- TAs are supposed to provide feedback within 1 week from the 1st submission deadline; for re-submissions within 1 week from the submission day (timings will be affected by holidays)

Module 2: Agenda

OW(Trapdoor)Functions

DH Key-Exchange

DL, CDH, DHH

Number Theory

RSA, ElGamal Cryptosystems

IND-CPA and IND-CCA

Digital Signatures

Secure Instant Messaging

- Security Notions
- The Signal Protocol
- Session Hijacking Attack

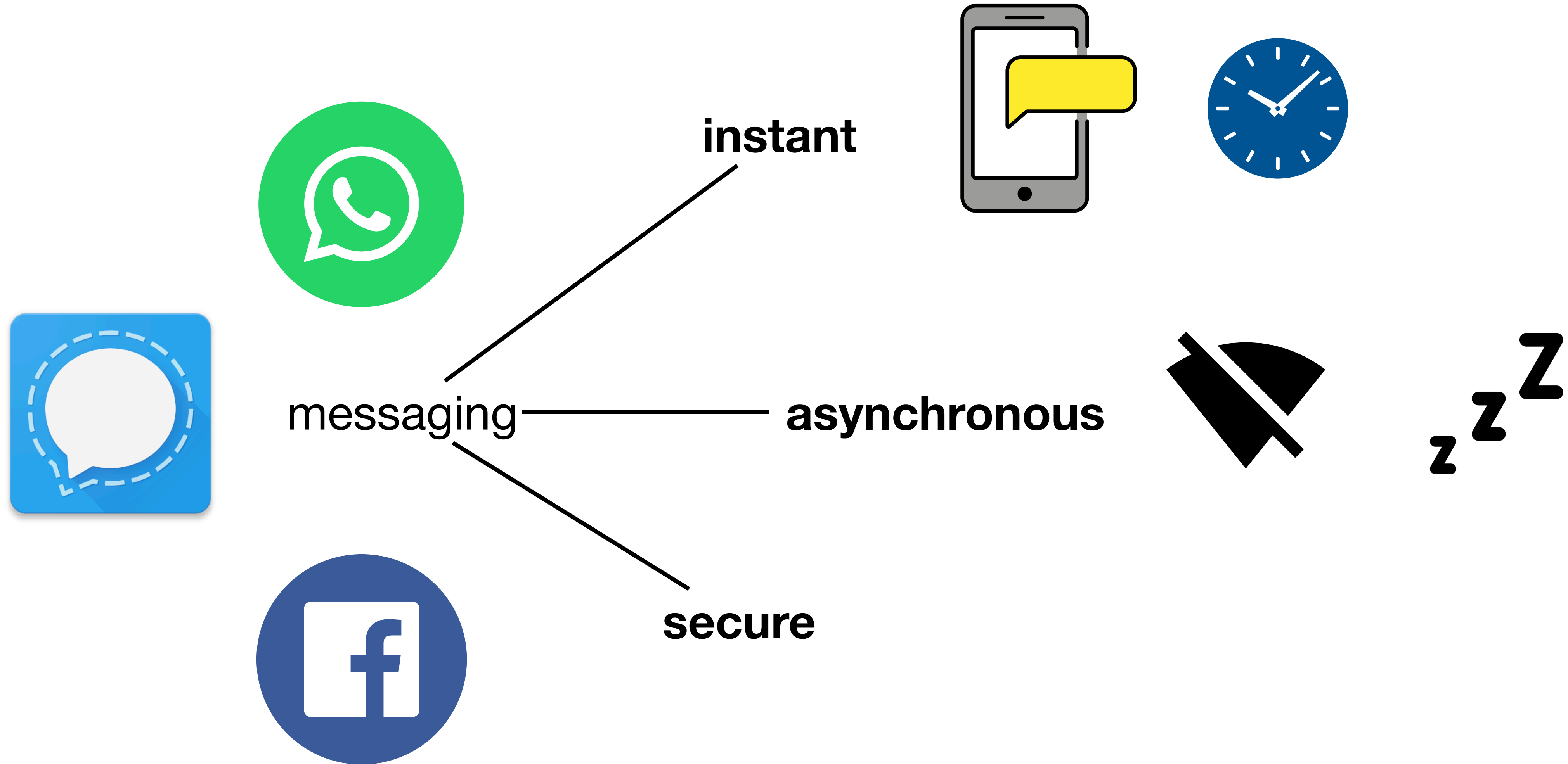
Post Quantum Cryptography

- The Lifespan of a Cryptosystem
- The State of Quantum Computers
- Landscape of PQC

Hash Functions

- Lamport PQ Secure Signature
- The Birthday Paradox - Proof

Signal: Privacy That Fits Your Pockets



Cryptographic Building Blocks

5.2. Recommended cryptographic algorithms

Taken from [here](#)

The following choices are recommended for instantiating the cryptographic functions from [Section 3.1](#):

- ***GENERATE_DH()***: This function is recommended to generate a key pair based on the **Curve25519** or Curve448 elliptic curves [7].
- ***DH(dh_pair, dh_pub)***: This function is recommended to return the output from the X25519 or X448 function as defined in [7]. There is no need to check for invalid public keys.
- ***KDF_RK(rk, dh_out)***: This function is recommended to be implemented using **HKDF [3] with SHA-256** or SHA-512 [8], using *rk* as HKDF *salt*, *dh_out* as HKDF *input key material*, and an application-specific byte sequence as HKDF *info*. The *info* value should be chosen to be distinct from other uses of HKDF in the application.
- ***KDF_CK(ck)***: **HMAC [2] with SHA-256** or SHA-512 [8] is recommended, using *ck* as the HMAC key and using separate constants as input (e.g. a single byte 0×01 as input to produce the message key, and a single byte 0×02 as input to produce the next chain key).
- ***ENCRYPT(mk, plaintext, associated_data)***: This function is recommended to be implemented with an **AEAD** encryption scheme based on either SIV or a composition of **CBC with HMAC** [5], [9]. These schemes provide some misuse-resistance in case a key is mistakenly used multiple times. A concrete recommendation based on

tiny variation of HMAC

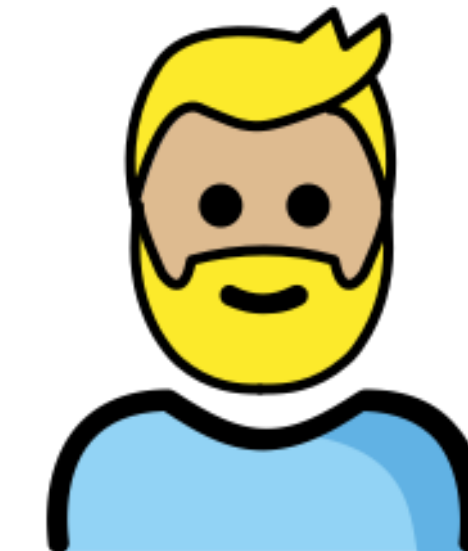
Signal: Privacy That Fits Your Pockets



secure messaging



Alice



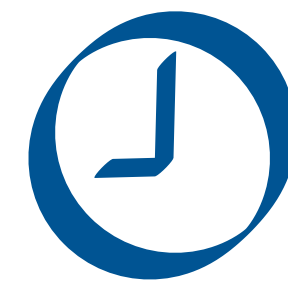
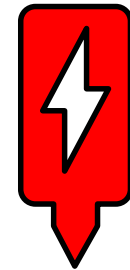
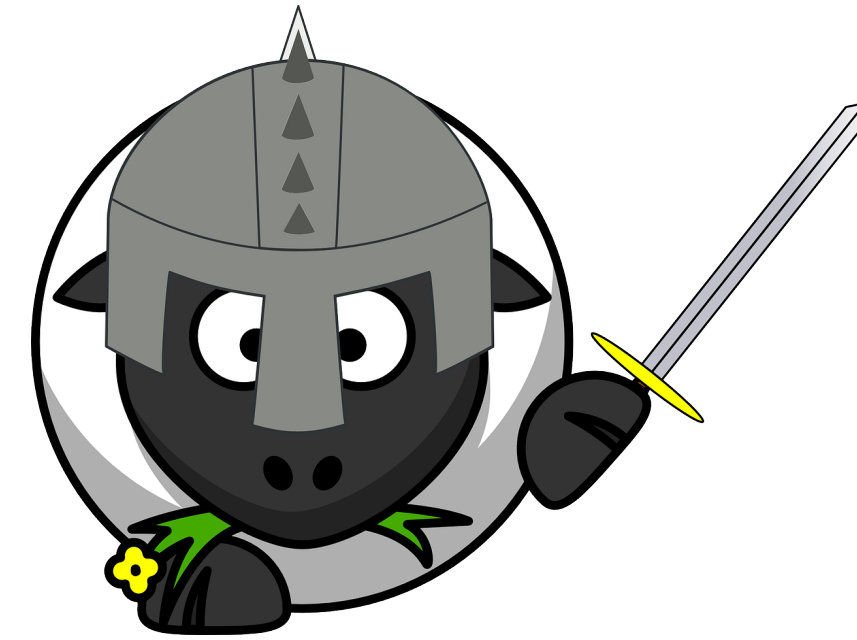
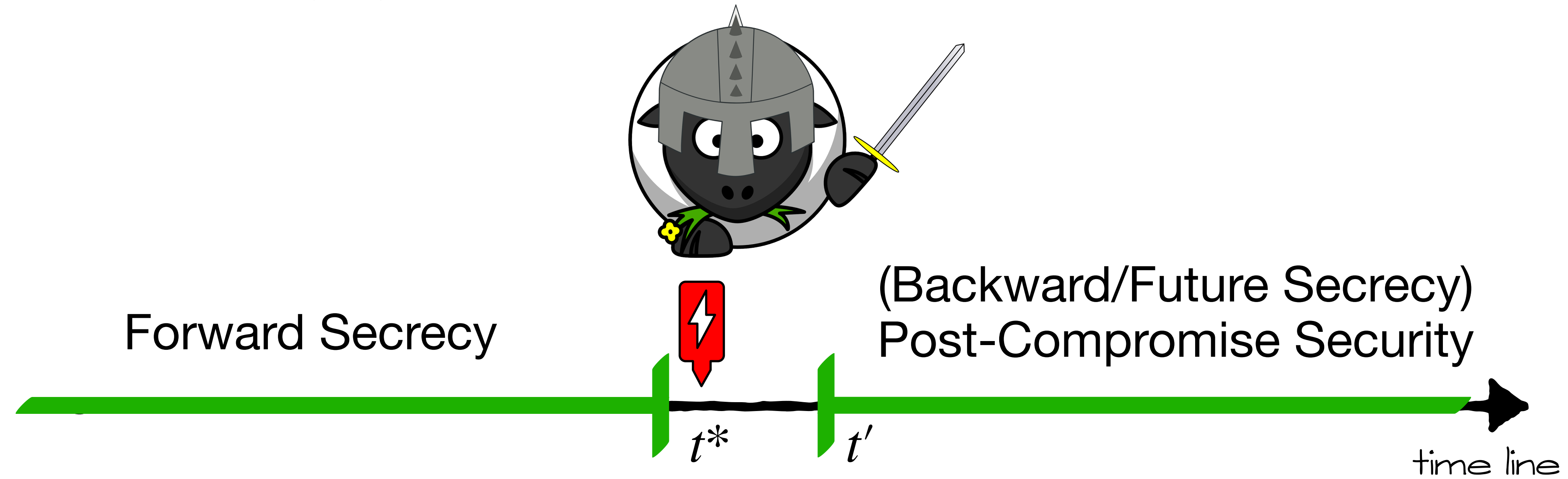
Bob

- end-to-end encryption
- interlocutor authentication
- message integrity

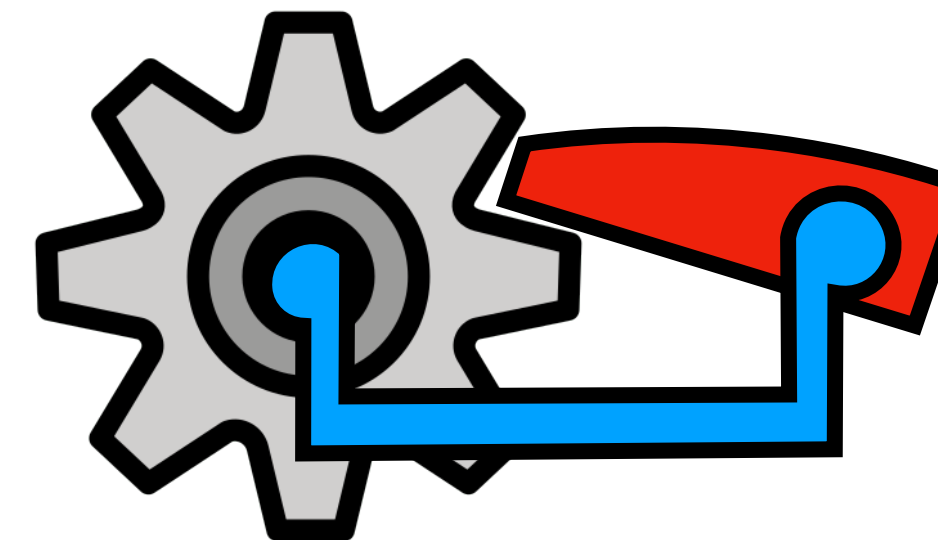
Signal: Privacy That Fits Your Pockets



secure messaging

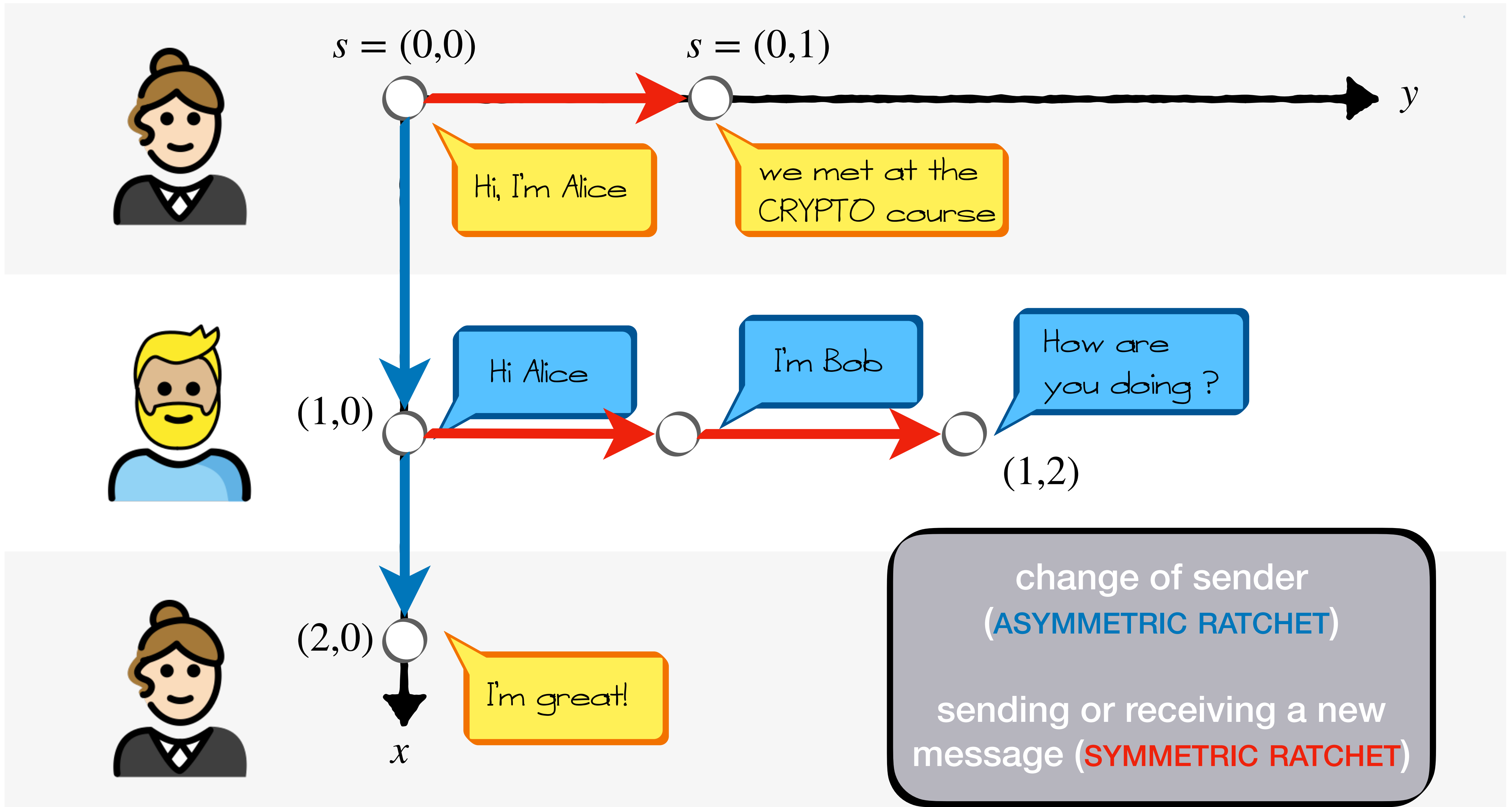
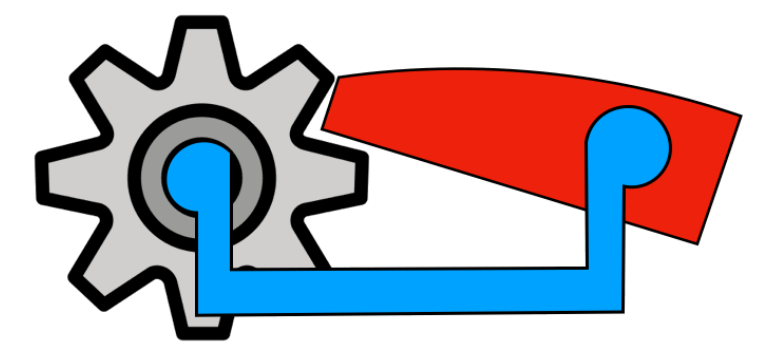


healing window

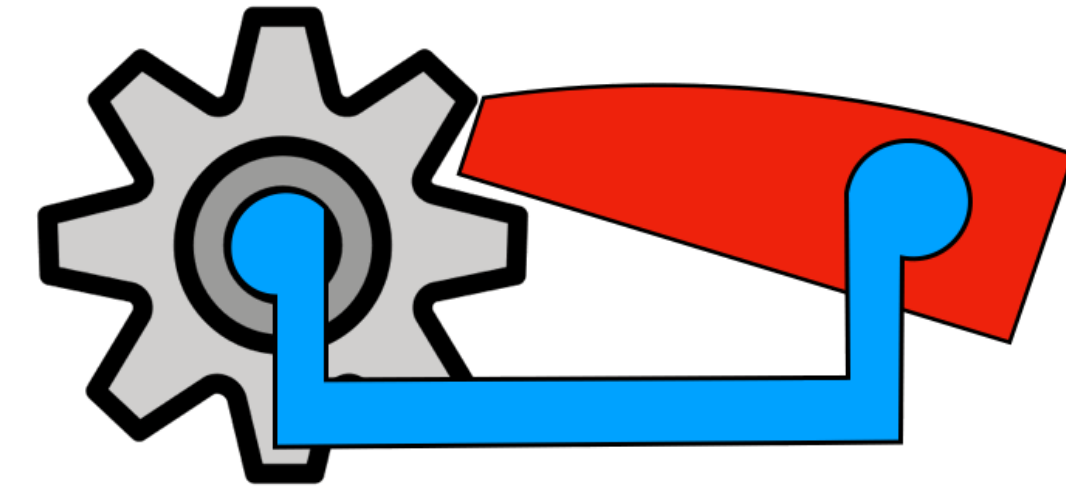


ratchet

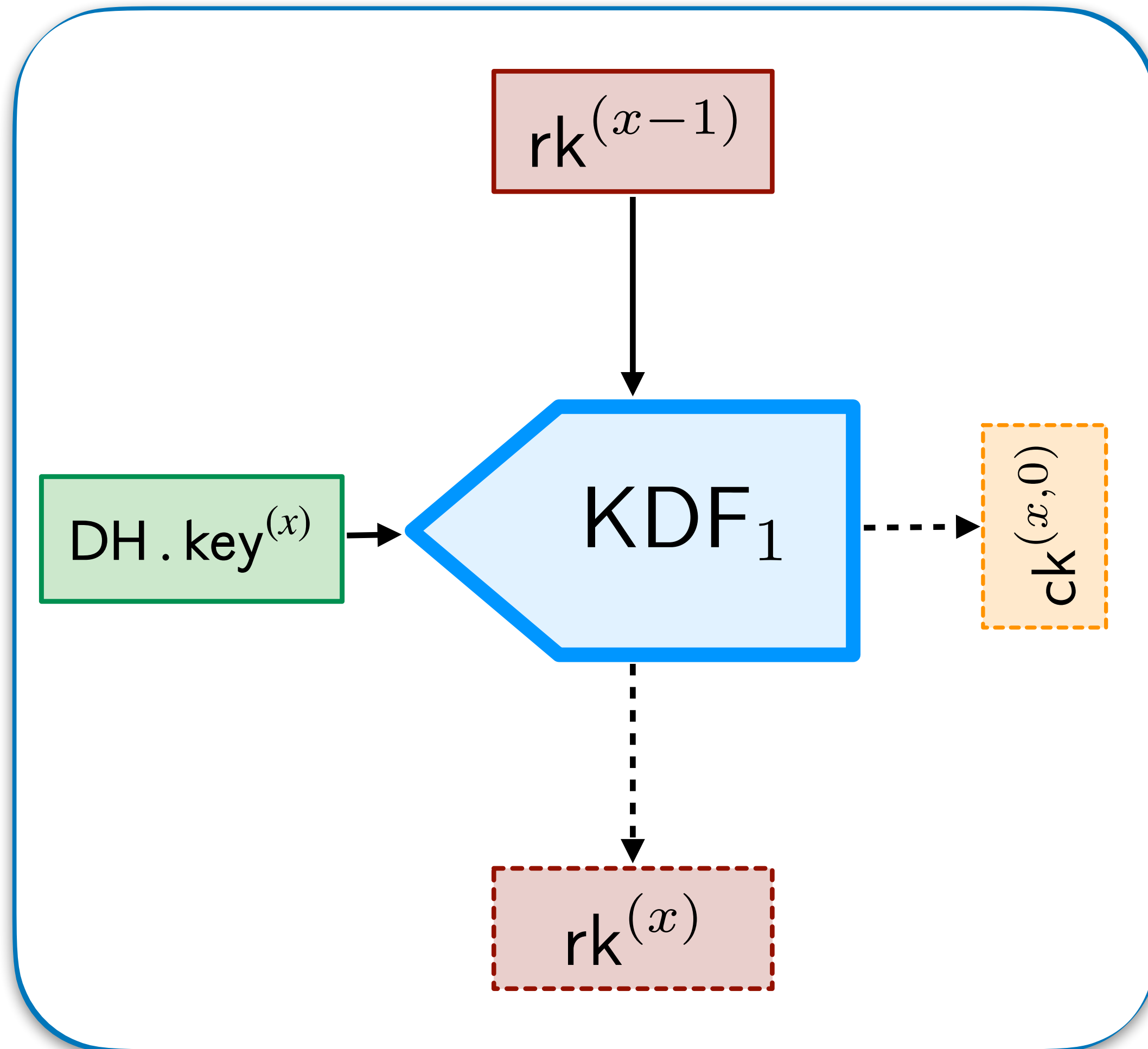
The Time Line of Asynchronous Messaging



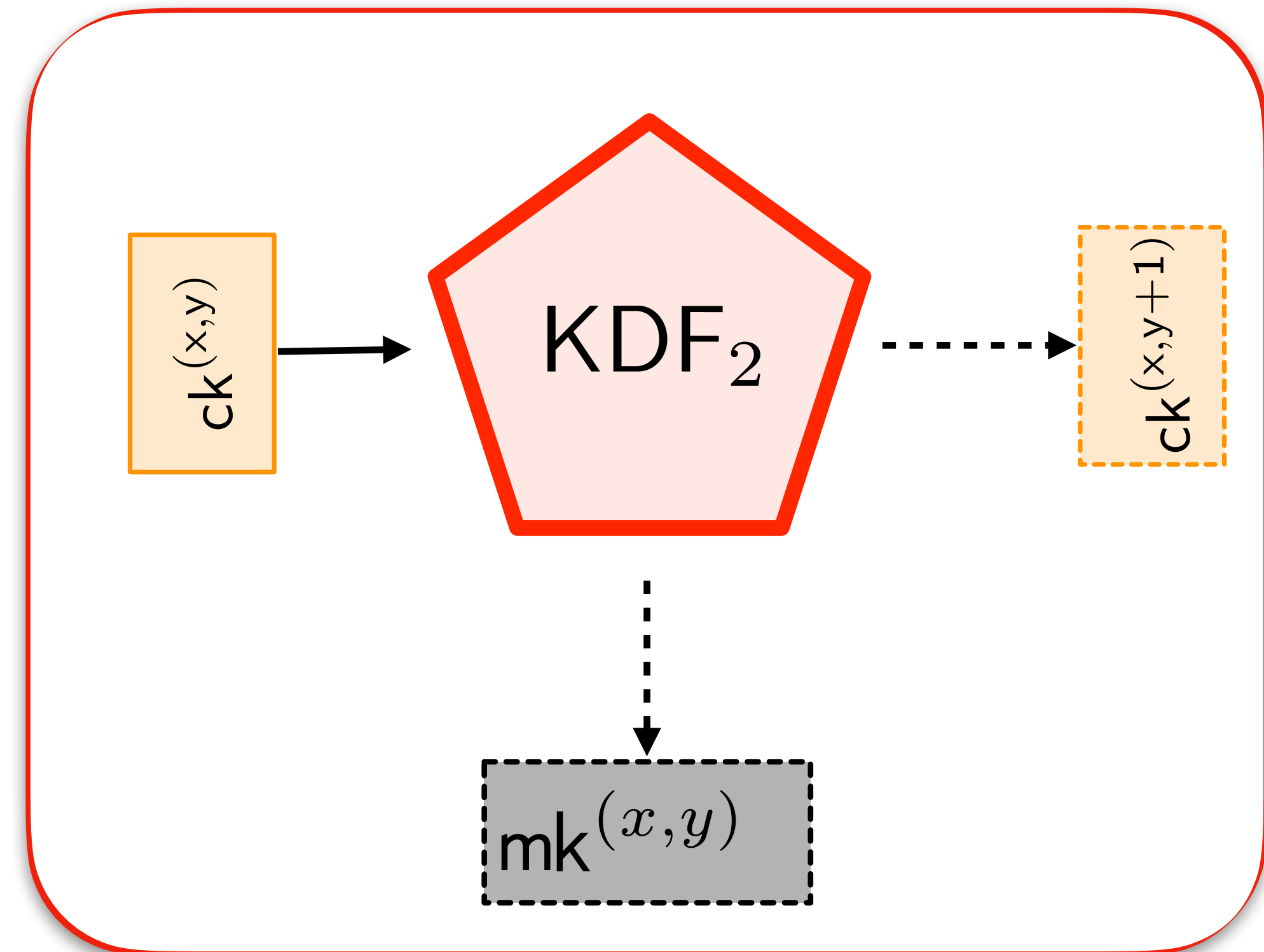
Asymmetric & Symmetric Ratcheting



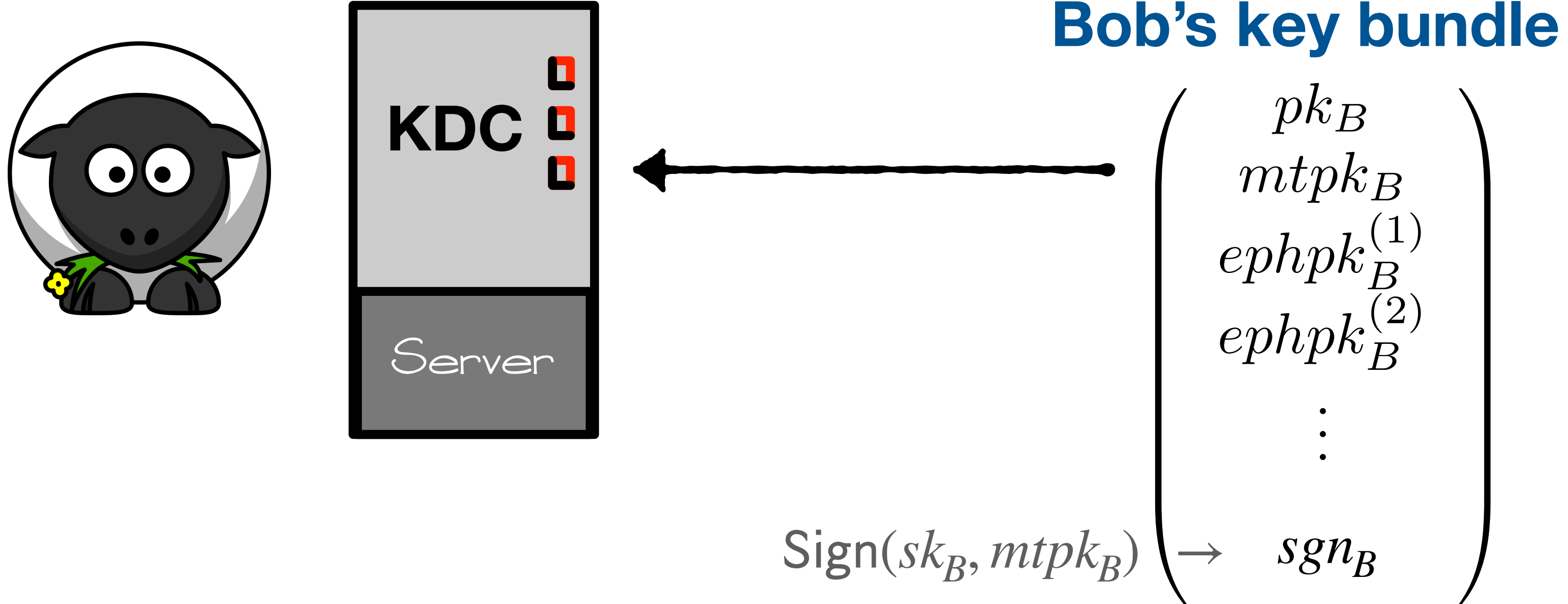
ASYMMETRIC RATCHET



SYMMETRIC RATCHET



Signal: Registration Phase



DH keys

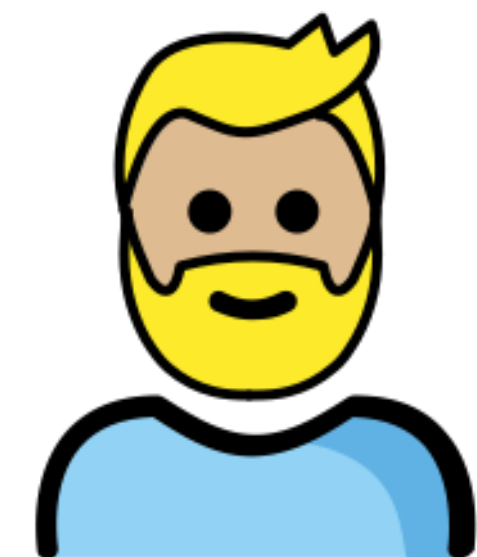
$$sk = x \leftarrow \mathbb{Z}_q$$

$$pk = g^x \in \mathbb{G}$$

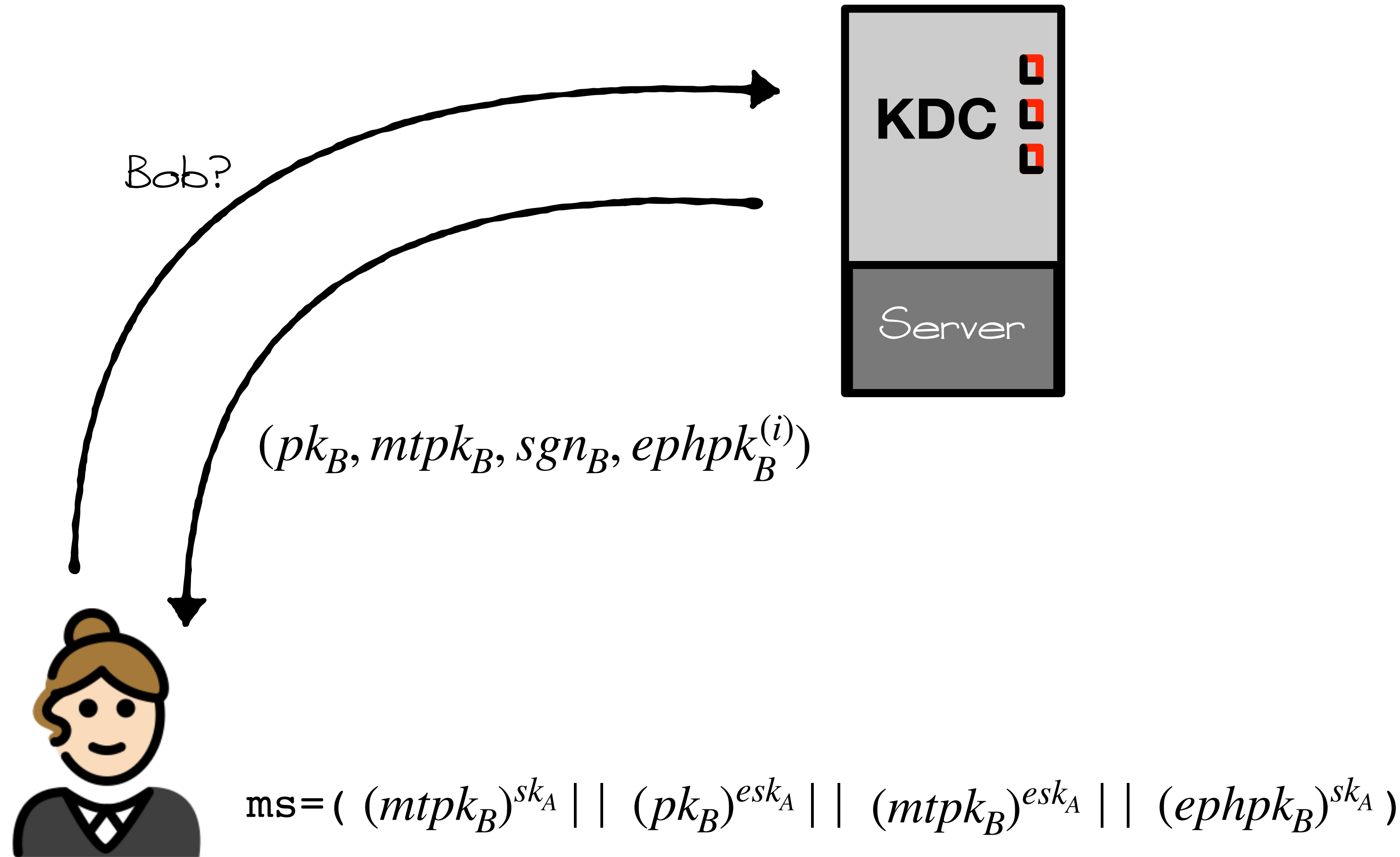
one long term identity key pair (pk_B, sk_B)

one medium term **prekey** pair $(mtpk_B, mtsk_B)$

multiple one-time **ephemeral** key pairs $\{(ephpk_B^{(i)}, ephsk_B^{(i)})\}_{i=1}^N$

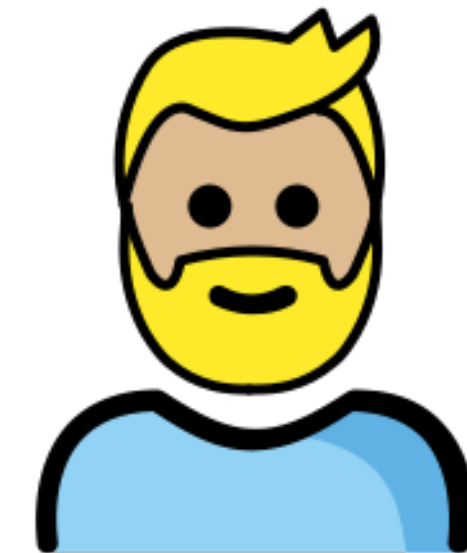


Signal: Session Setup Phase



Bob's key bundle

$$\left(\begin{array}{c} pk_B \\ mtpk_B \\ ephpk_B^{(1)} \\ ephpk_B^{(2)} \\ \vdots \\ sgn_B \end{array} \right)$$



sk_A identity key
 $mtsk_A$ prekey
 $\{ephsk_A^{(i)}\}$ ephemerals

fresh ephemeral key pair

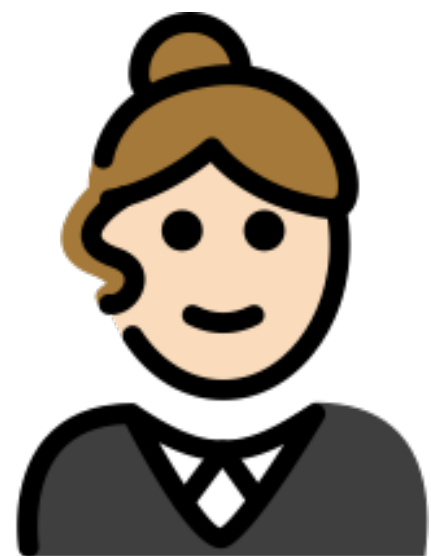
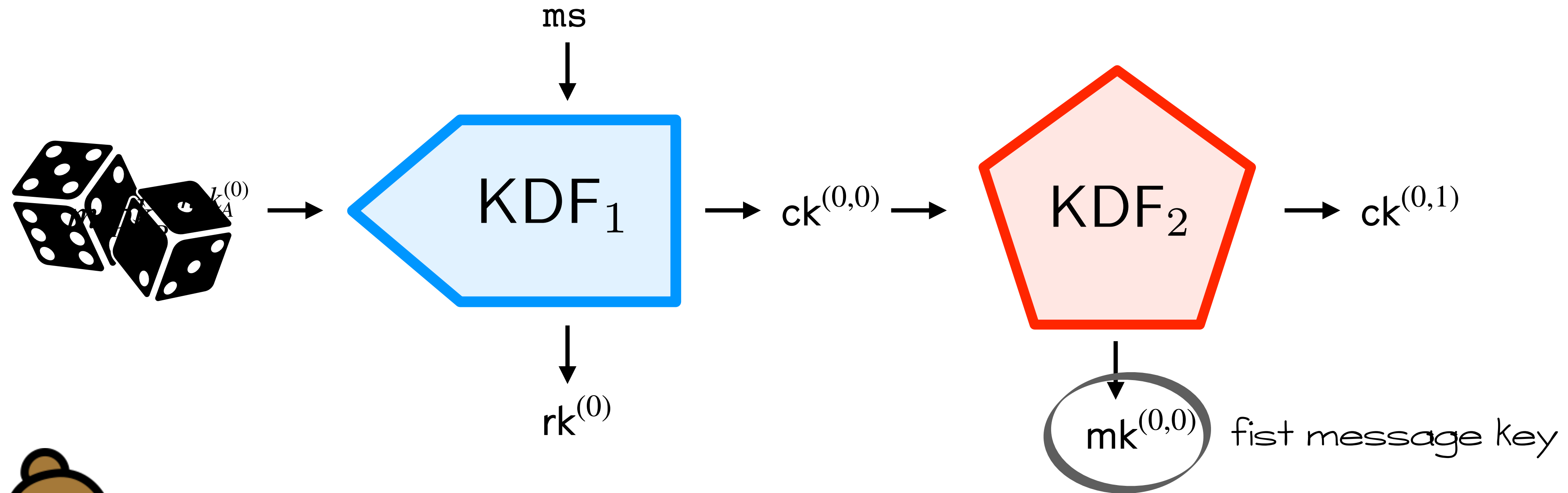
$$esk_A = x \leftarrow \$ - \mathbb{Z}_q$$

$$epk_A = g^x \in \mathbb{G}$$

(pk_B, sk_B)
 $(mtpk_B, mtsk_B)$
 $\{(ephpk_B^{(i)}, ephsk_B^{(i)})\}_{i=1}^N$

Signal: Asymmetric Ratchet

... And Symmetric Ratchet



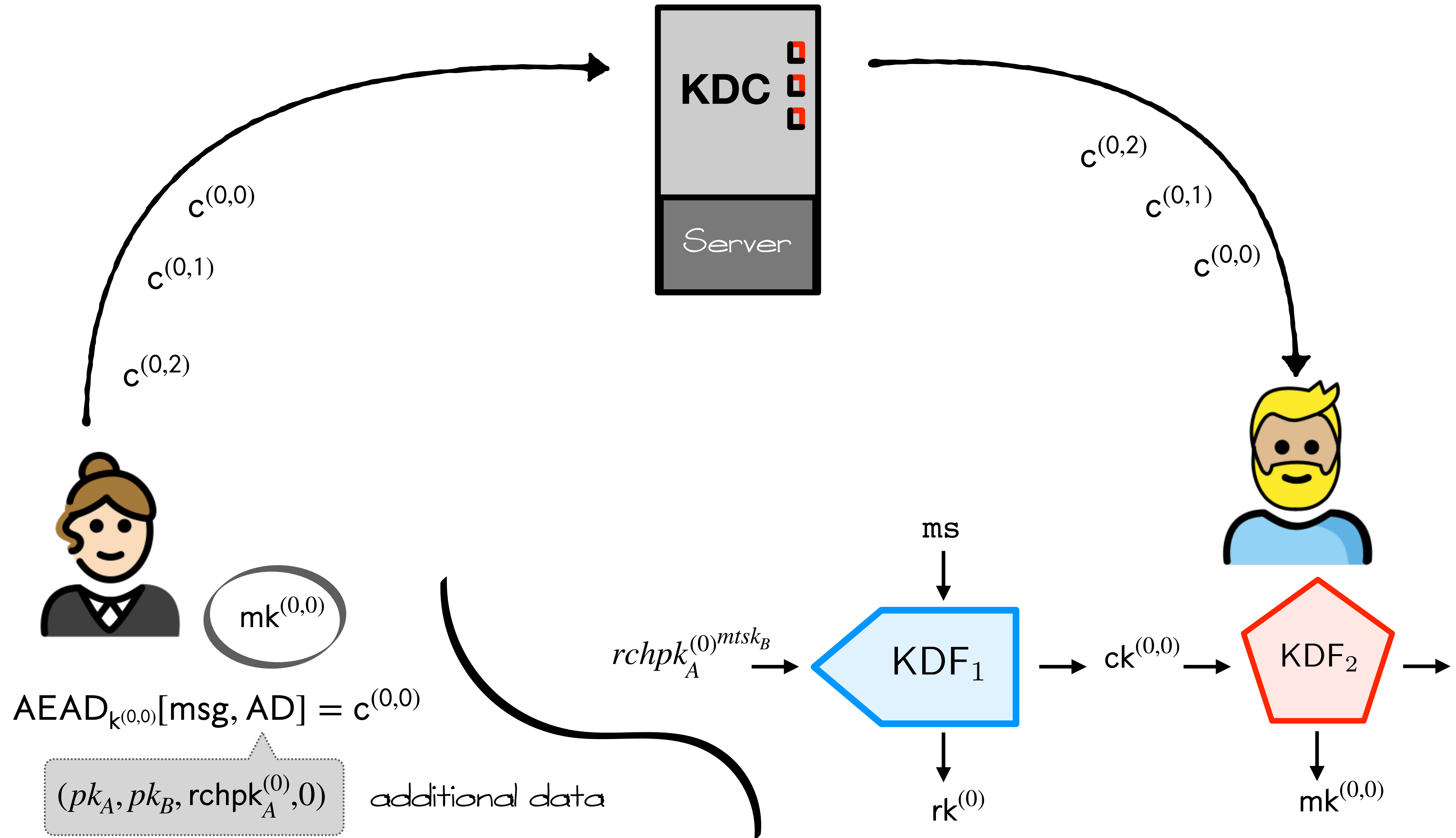
$$ms = ((mtpk_B)^{sk_A} || (pk_B)^{esk_A} || (mtpk_B)^{esk_A} || (ephpk_B)^{sk_A})$$

fresh ratchet key pair

$$rchsk_A = x \leftarrow \mathcal{S} - \mathbb{Z}_q$$

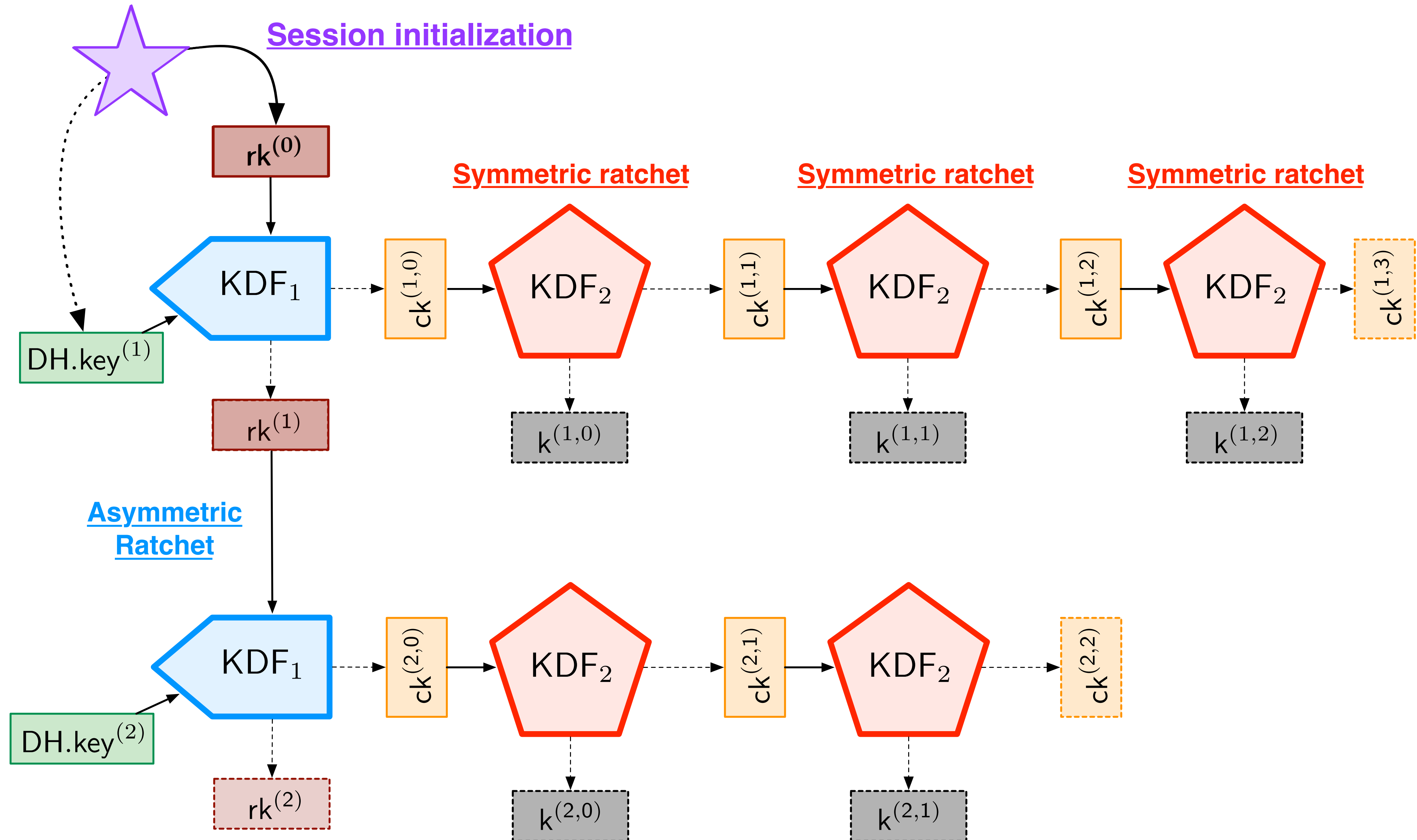
$$rchpk_A = g^x \in \mathbb{G}$$

Signal: Messaging

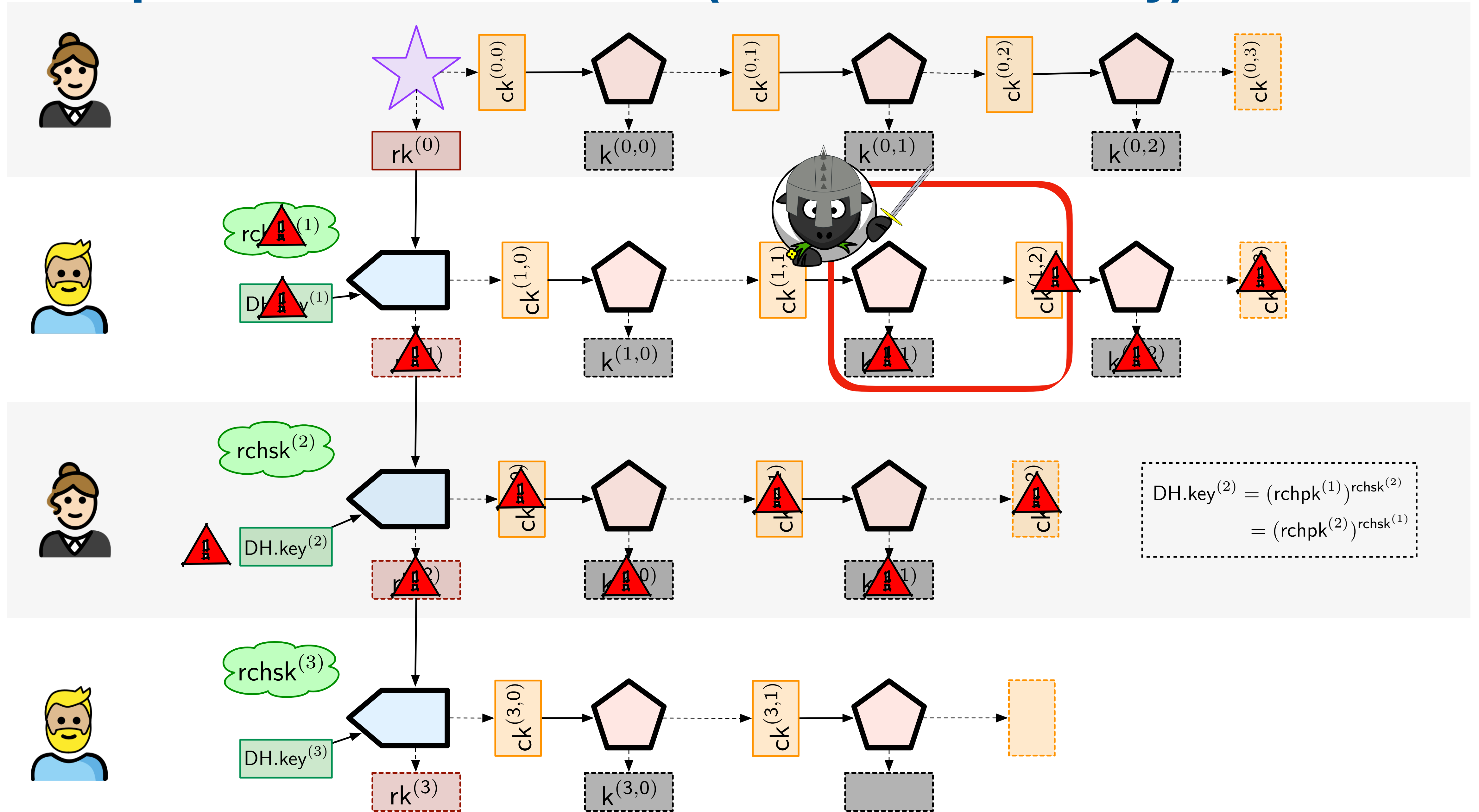


Two Layers of Ratchets

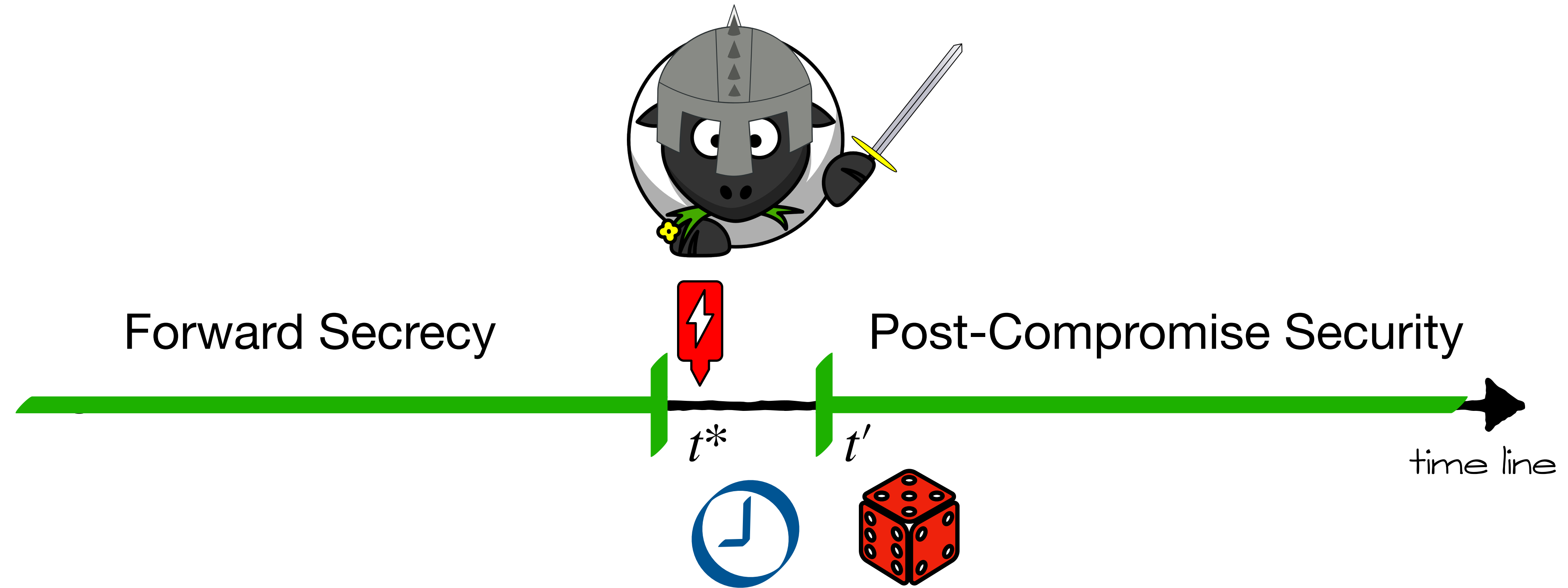
Note: from now on the message key mk is denoted as k



The Impact of Reveal Attacks (Passive Adversary)

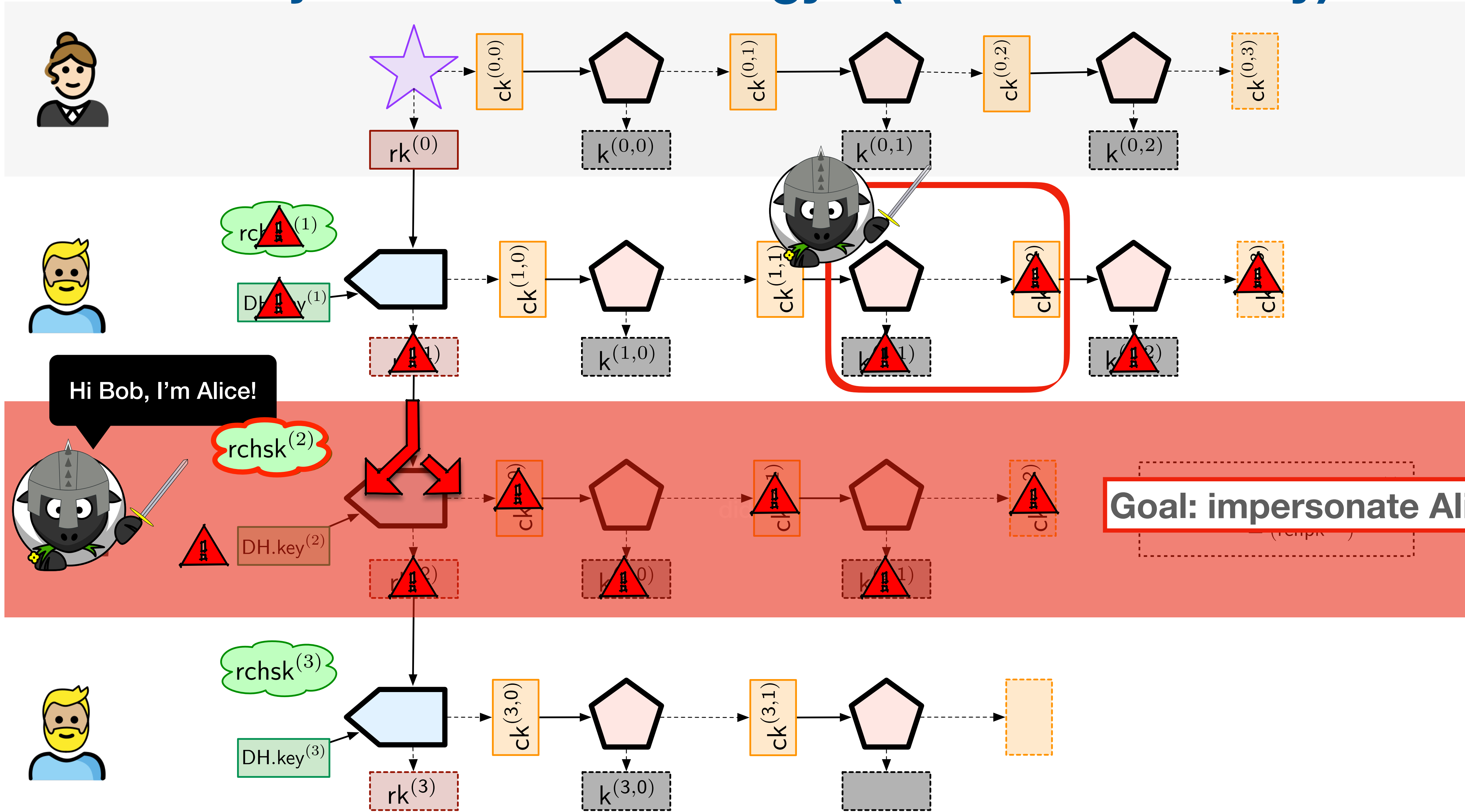


Healing Window for Reveal Attacks

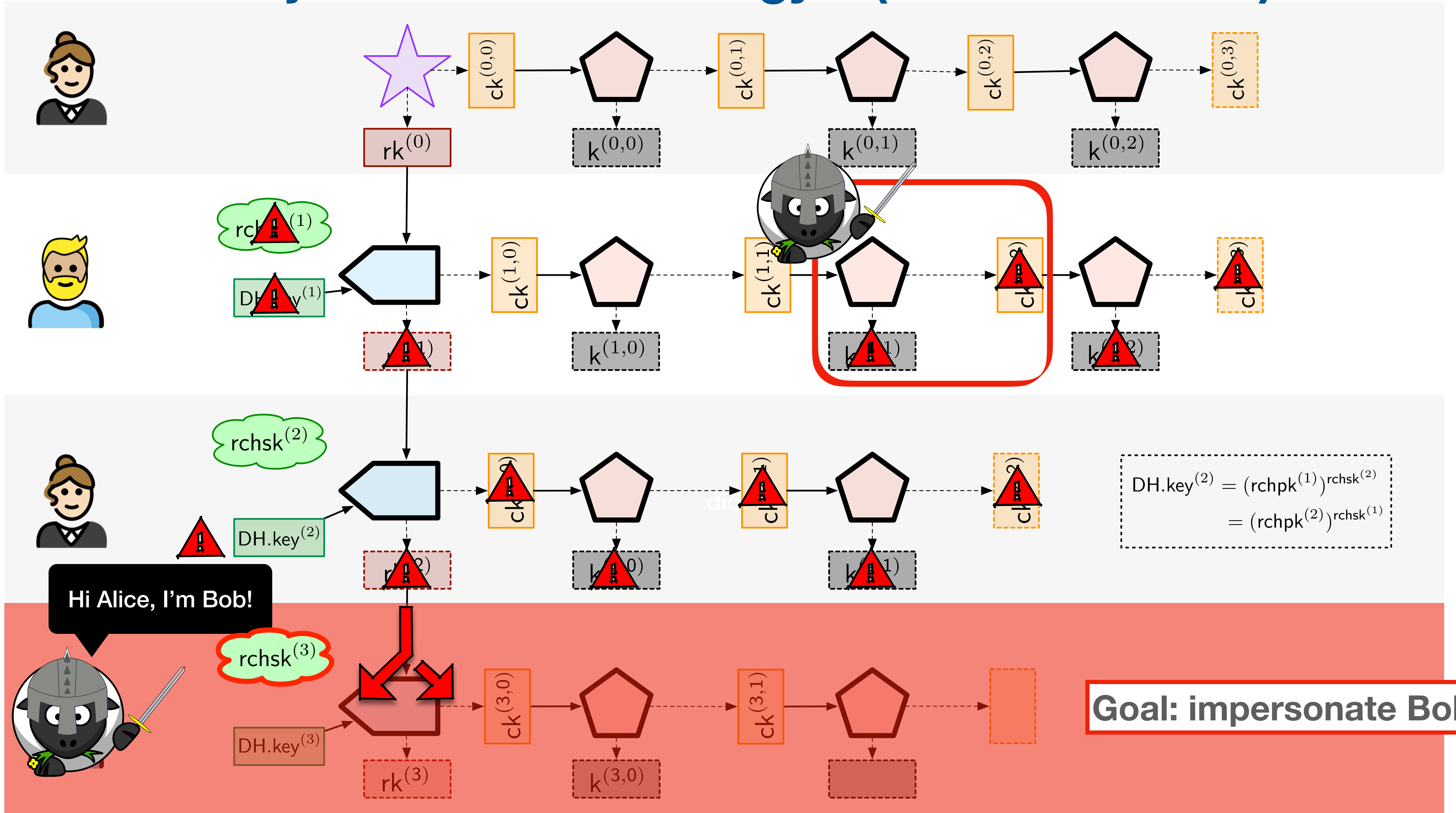


healing window
= 2 asymmetric ratchets

A Reveal & Hijack Attack - Strategy 1 (Active Adversary)



A Reveal & Hijack Attack - Strategy 2 (Active Attacker)



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DL, CDH, DHH

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Post Quantum Cryptography

- The Lifespan of a Cryptosystem
- The State of Quantum Computers
- Landscape of PQC

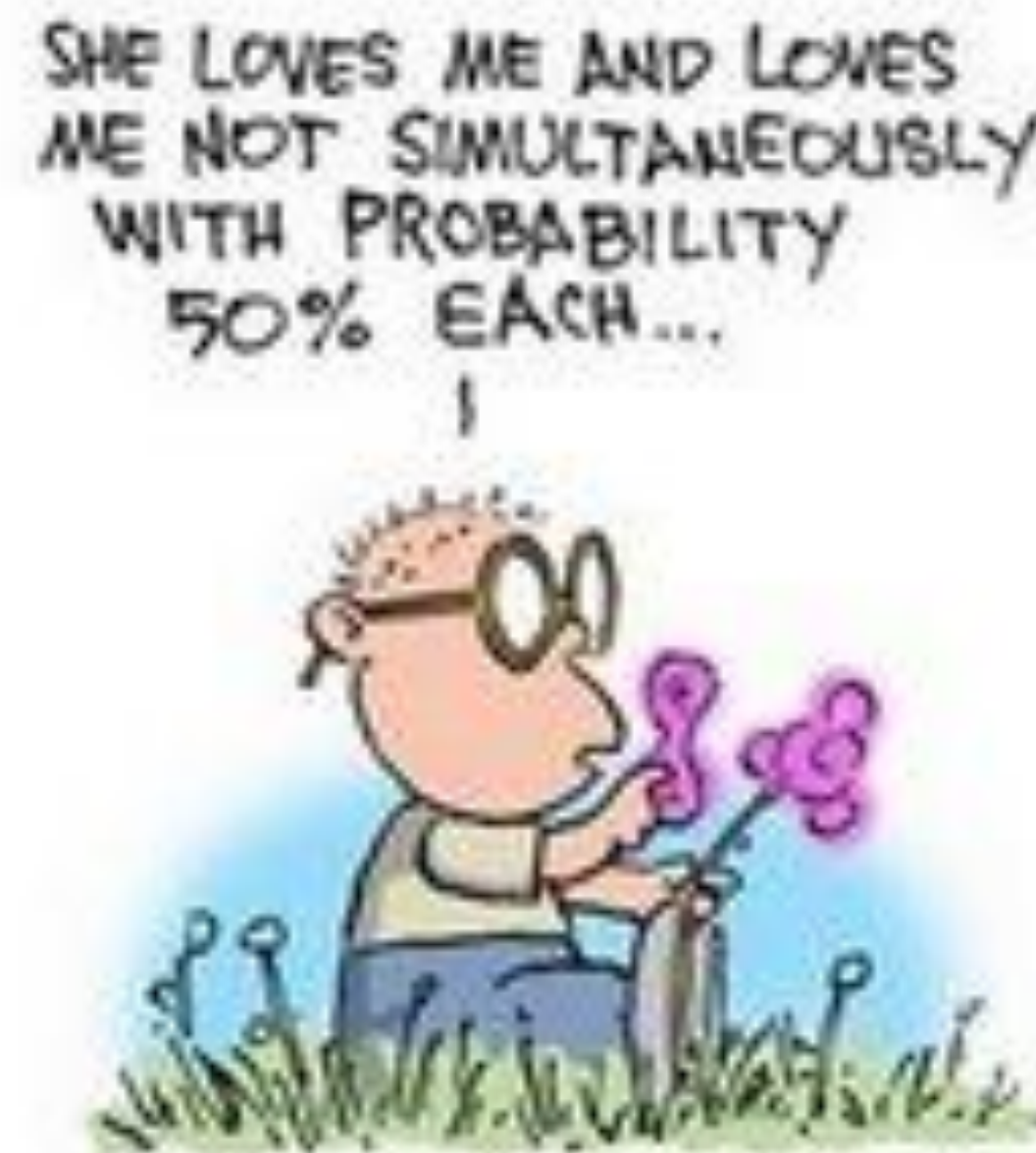
Hash Functions

- Lamport PQ Secure Signature
- The Birthday Paradox - Proof

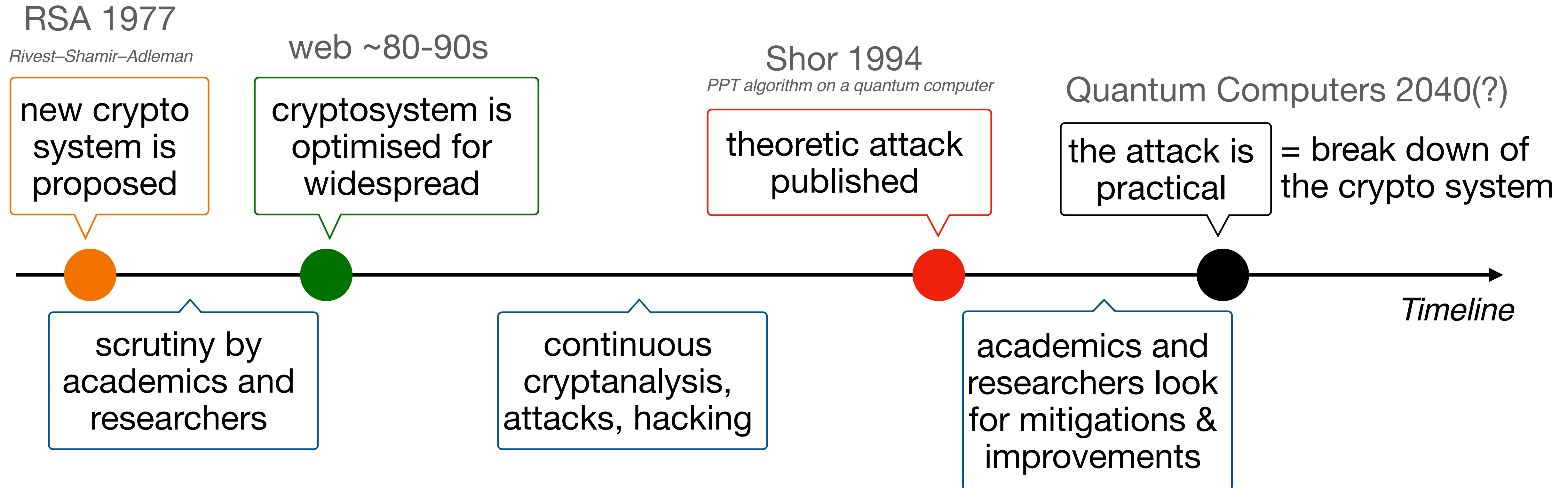
Classical Vs Quantum Computers

The security of **classical** crypto relies on the intractability of certain problems using **modern** computers.

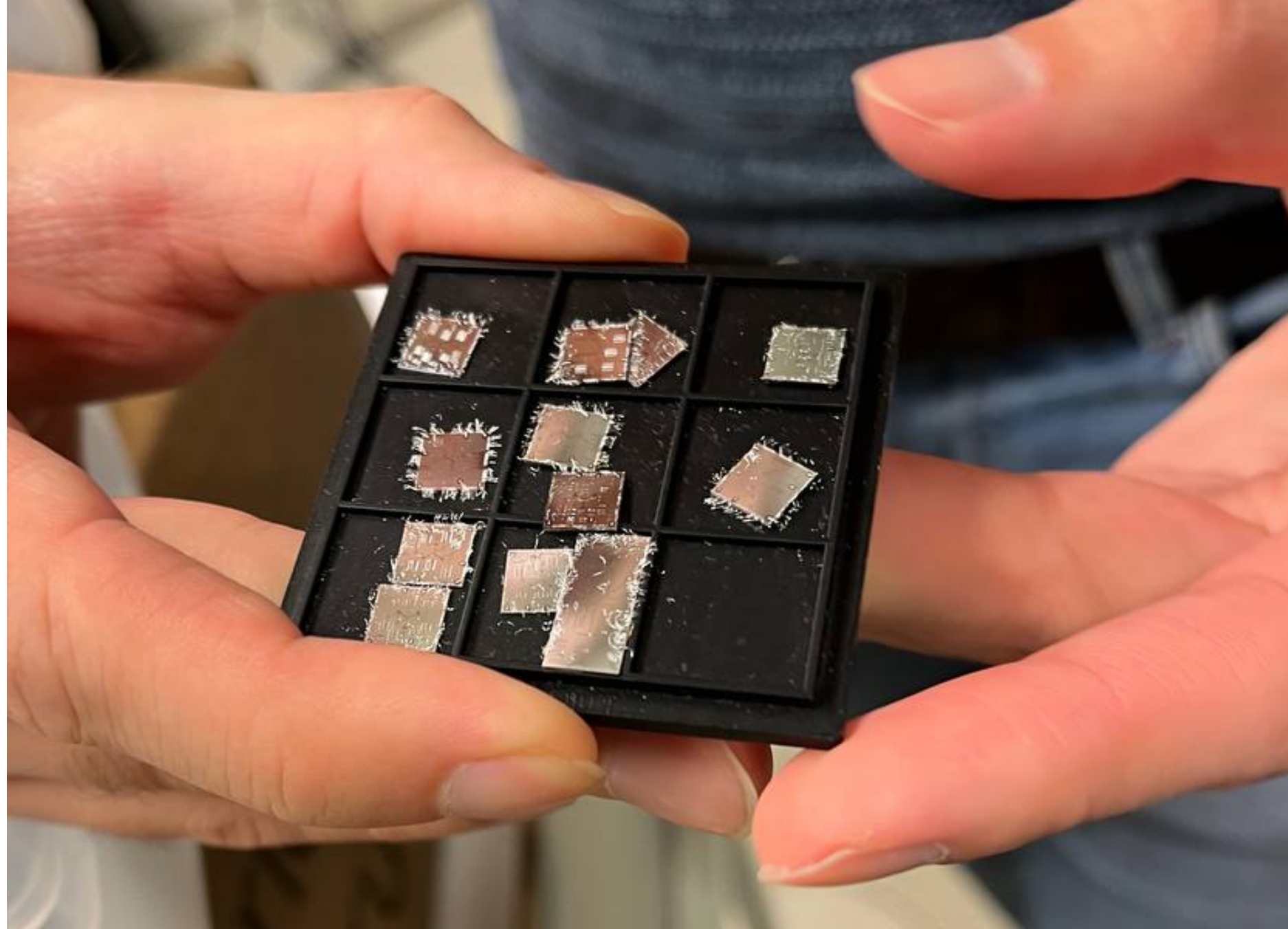
The security of **post-quantum** crypto relies on the intractability of certain problems using **quantum** computers.



Time Line of 'Secure' Cryptographic Algorithms



The State of Quantum Computers



200 seconds QC
= 10,000 years CC

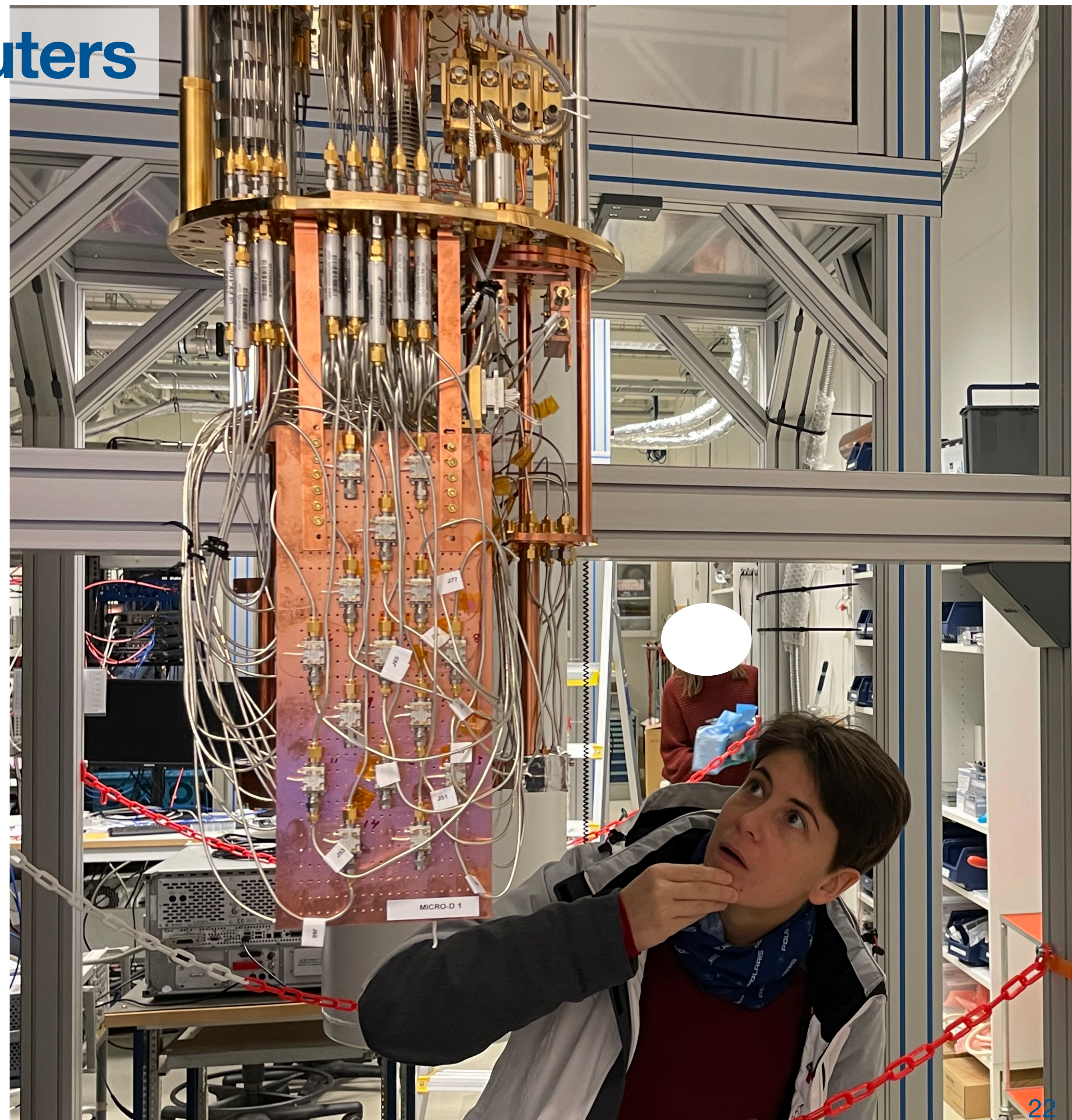
@Chalmers: 25 qubits

Google: Sycamore chip, 54 qubits

IBM: 433 qubits

NSA: ?

For fun: [thequbitgame](https://thequbitgame.com)



Cryptosystems Lifespan

the best known attack takes exponential time/space

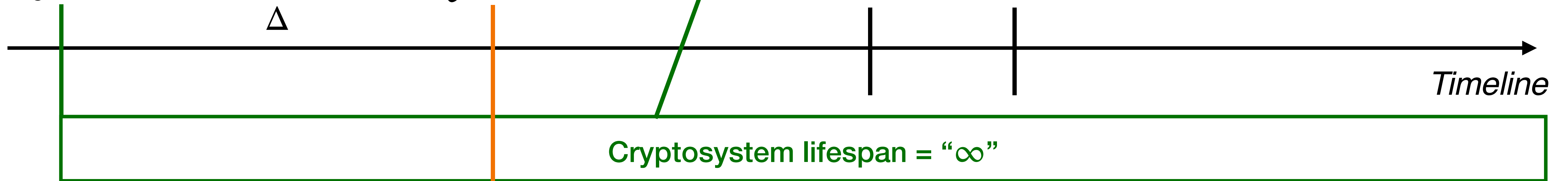
in y years the best known attack will run in time x

Shor 1994

Classical crypto broken in PPT algorithm on a quantum computer

migrate/transition to a new cryptosystem

Post-Quantum Cryptography



Cryptosystem lifespan = $\Delta + y + x$

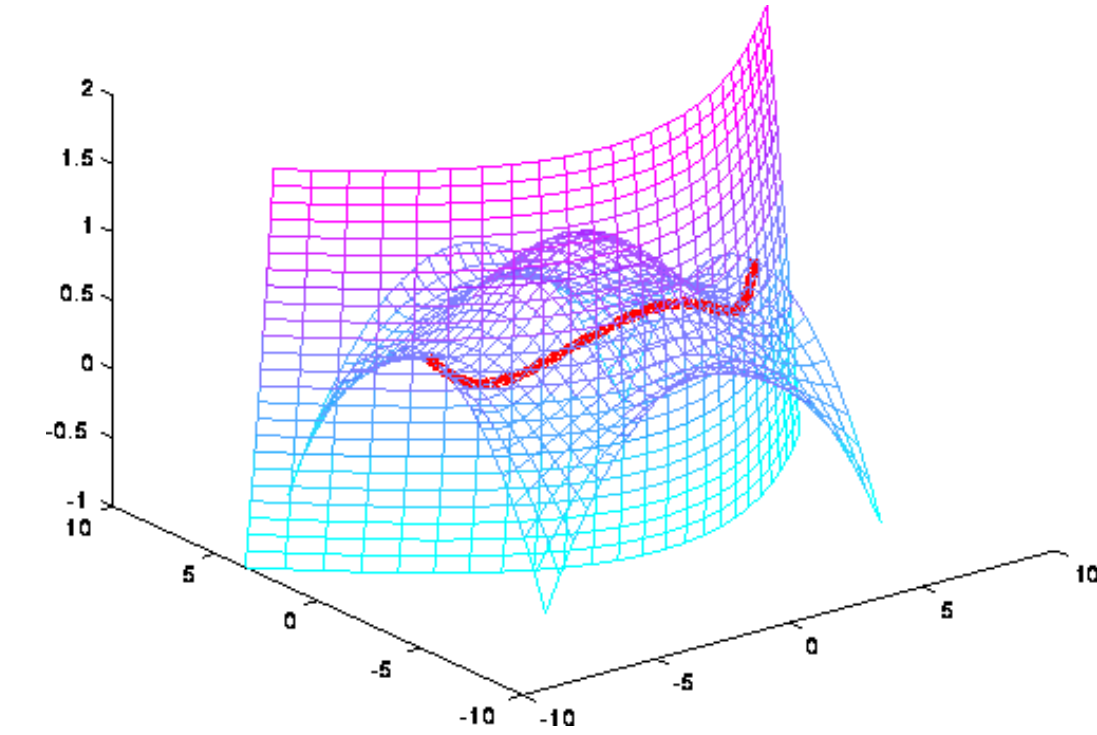
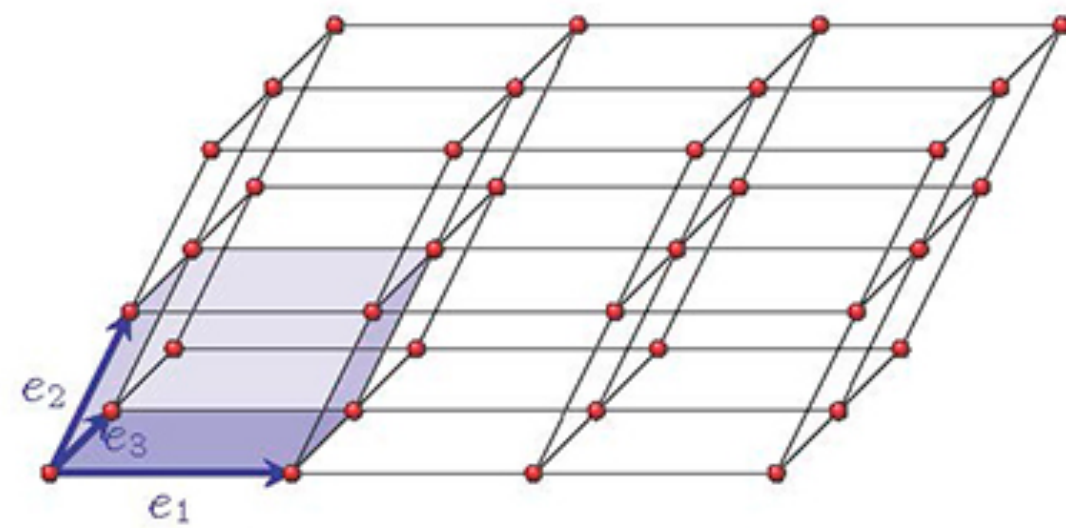
initial cryptosystem is completely insecure

Post Quantum (PQ) Security

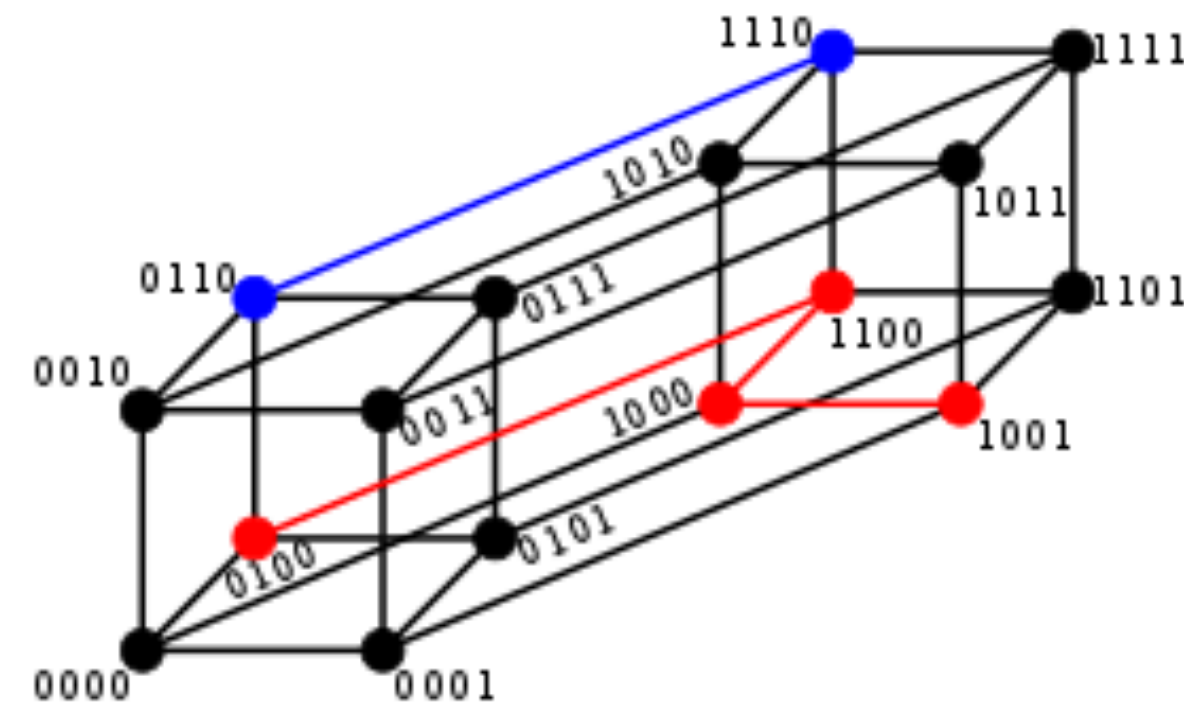
For more details check [Tanja Lange's lectures](#)

Multi Variate Quadratic Equations

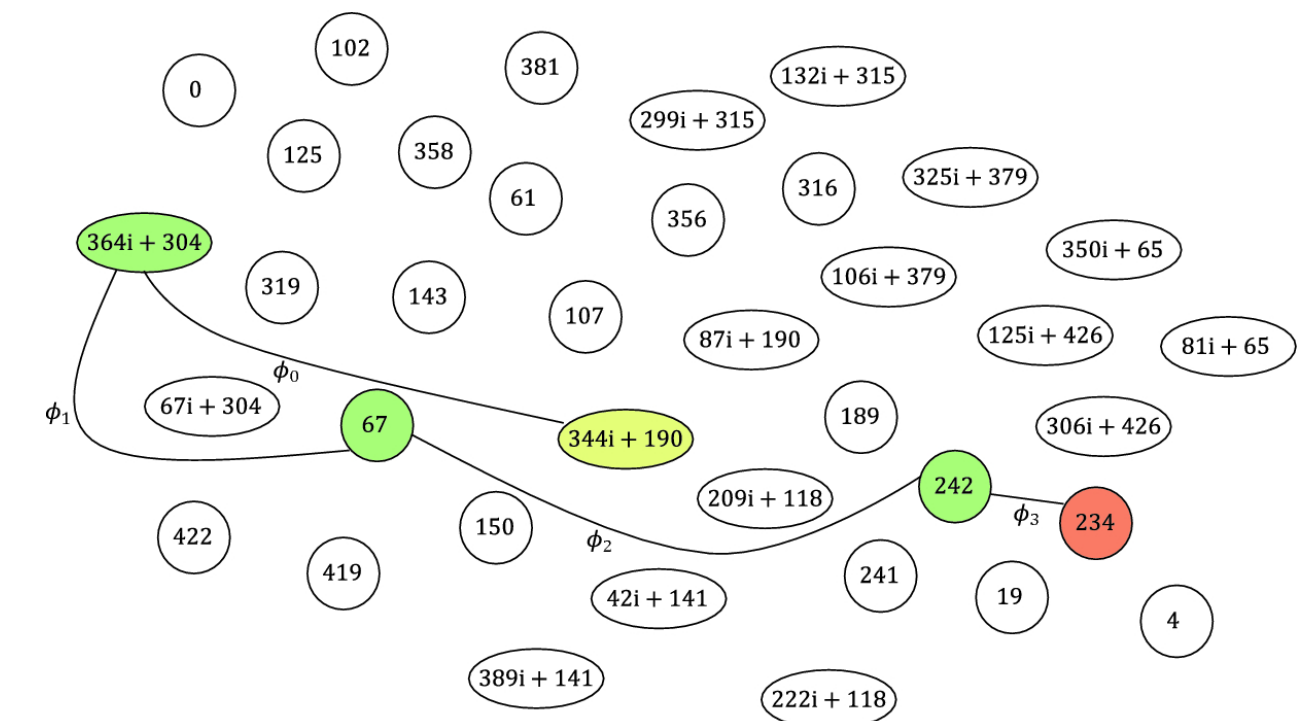
Lattices



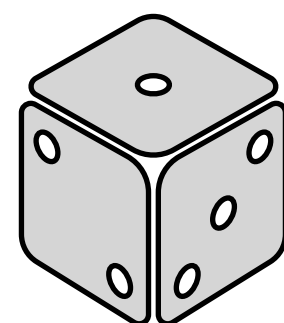
Codes



Isogenies



Hash Functions



Information Theoretic

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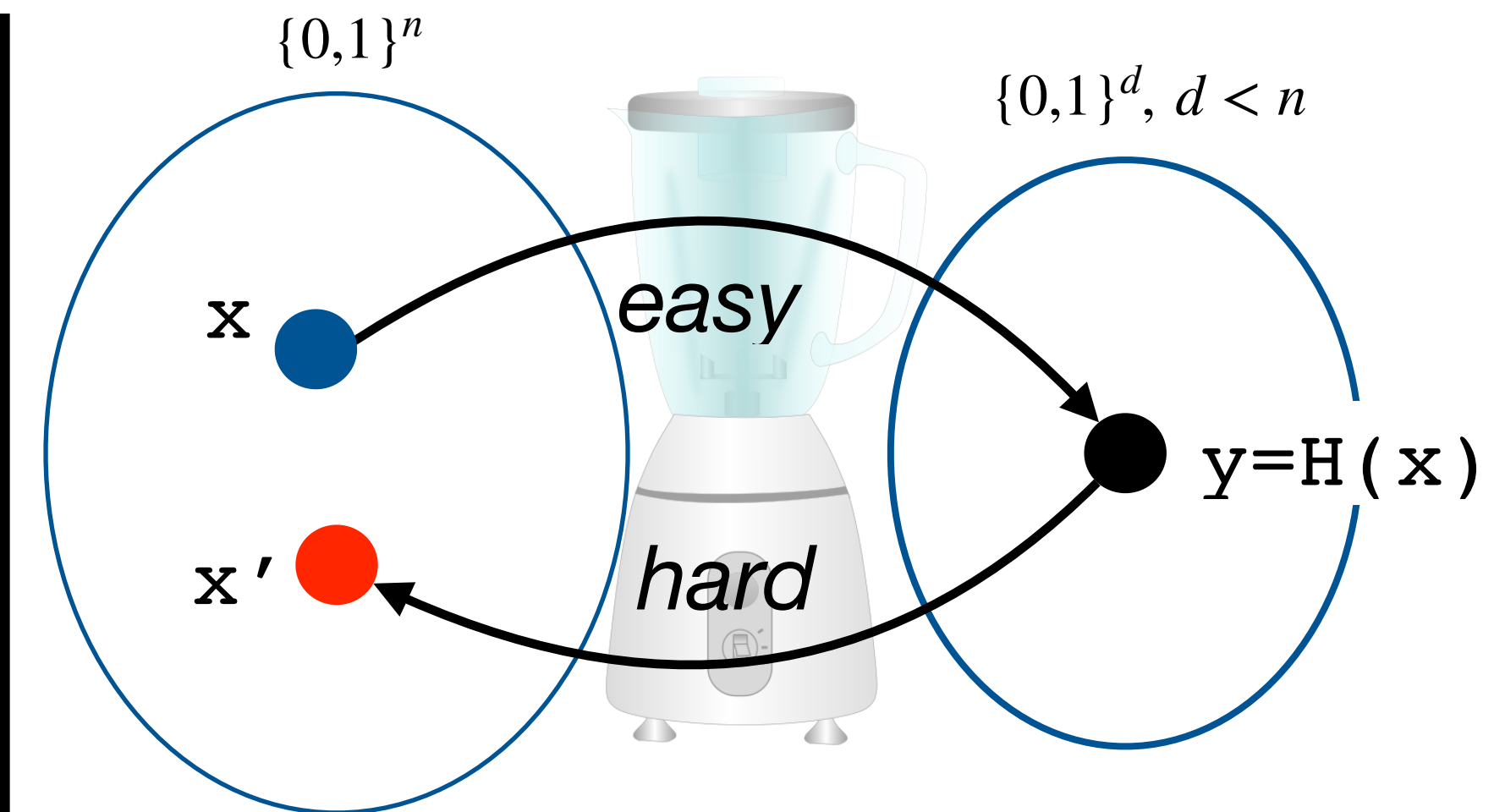
- Lamport PQ Secure Signature
- The Birthday Paradox - Proof

Lamport Signature (Hash-Based PQ)

KeyGen $(n) \Rightarrow (sk, pk)$

$$sk = \begin{pmatrix} x_{1,0} & x_{2,0} & \dots & x_{v,0} \\ x_{1,1} & x_{2,1} & \dots & x_{v,1} \end{pmatrix} \leftarrow \$X^{2v}$$

$$pk = \begin{pmatrix} y_{1,0} & y_{2,0} & \dots & y_{v,0} \\ y_{1,1} & y_{2,1} & \dots & y_{v,1} \end{pmatrix} \text{ where } y_{i,b} = f(x_{i,b})$$



Sign $(sk, m) \Rightarrow \sigma$

Take the bit decomposition of m

$$\sigma = (x_{1,m[1]}, x_{v,m[v]}, \dots, x_{v,m[v]})$$

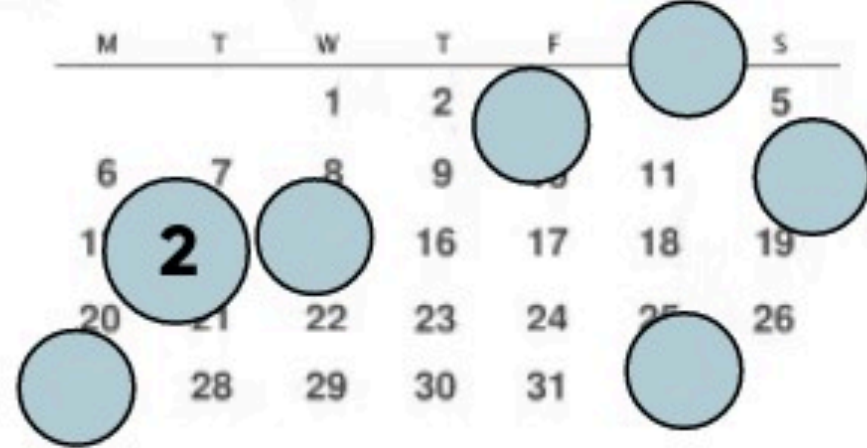
$f: X \rightarrow Y$ is a one-way function and messages are in $m \in \{0,1\}^v$

Ver $(pk, m, \sigma) \Rightarrow \{0, 1\}$

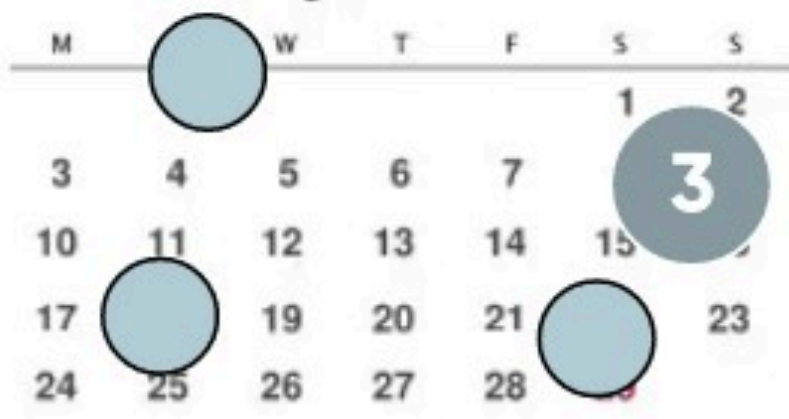
Accept iff $f(\sigma_{i,m[i]}) = y_{i,m[i]}$ for all $i \in \{1,2,\dots,v\}$

The Birthday Paradox

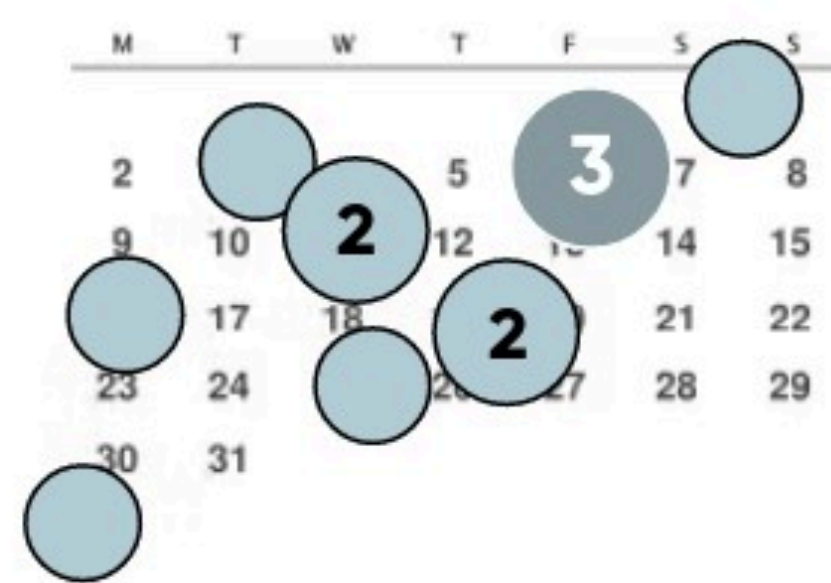
January



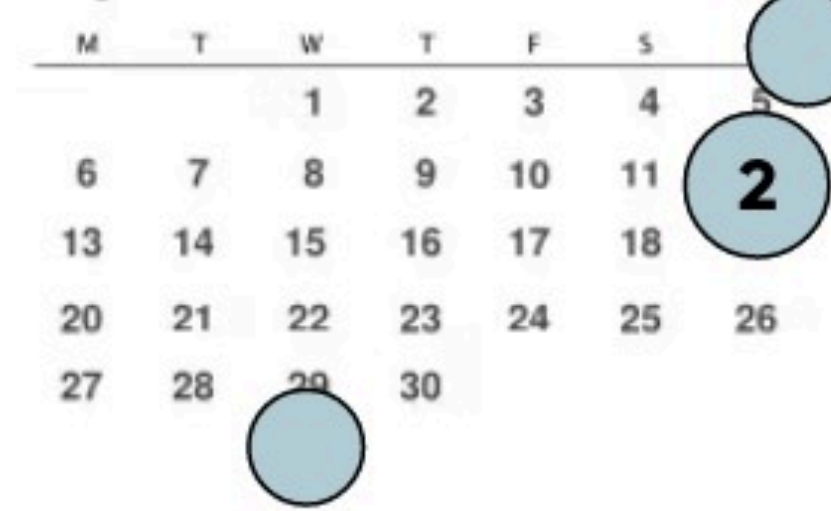
February



March



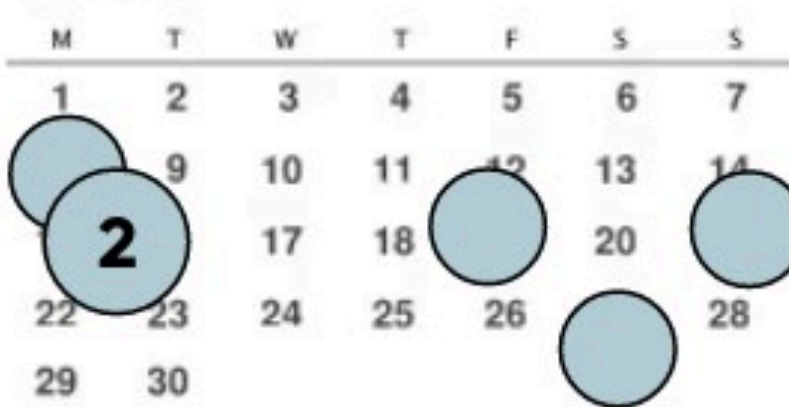
April



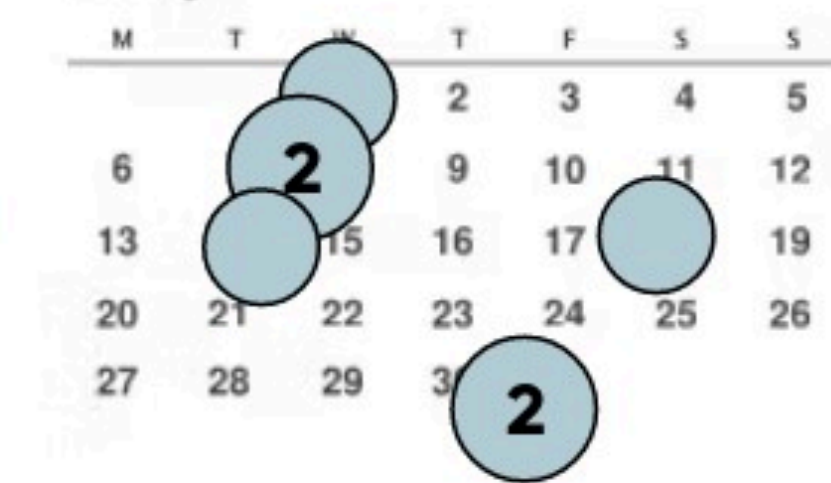
May



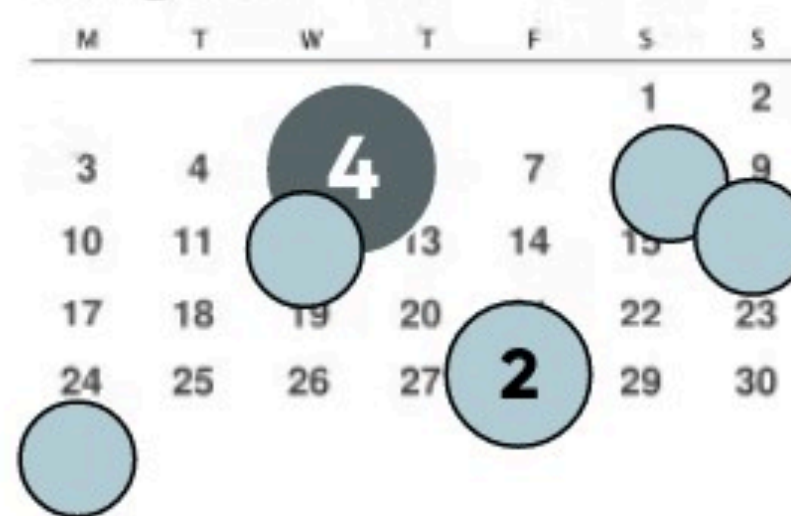
June



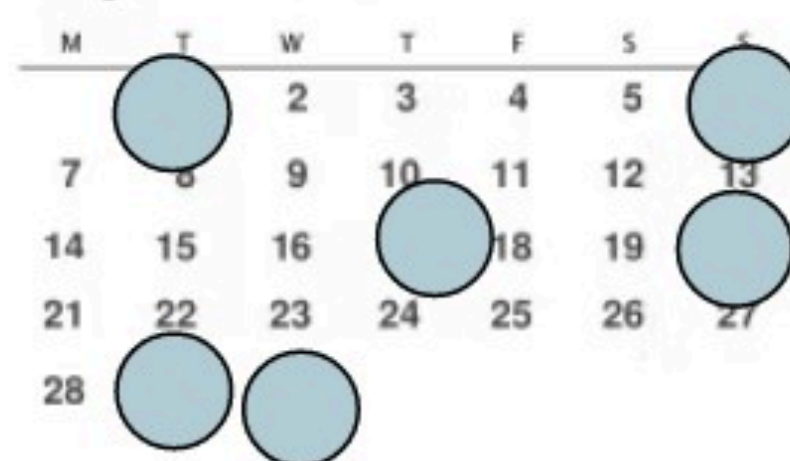
July



August



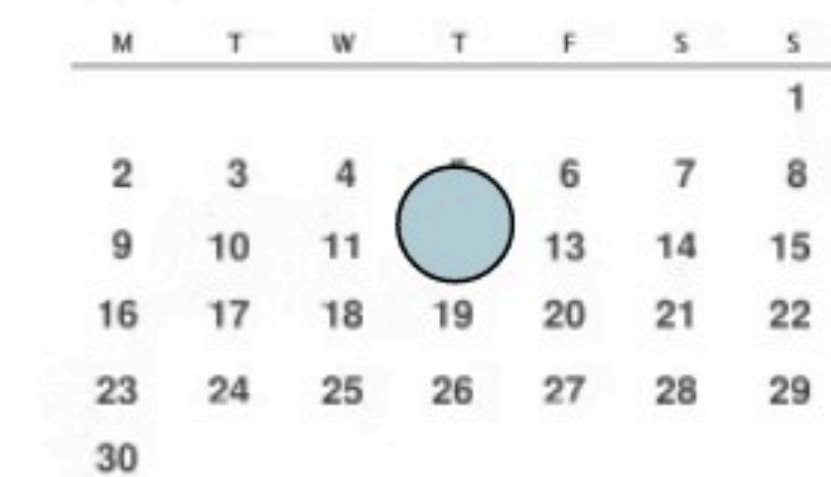
September



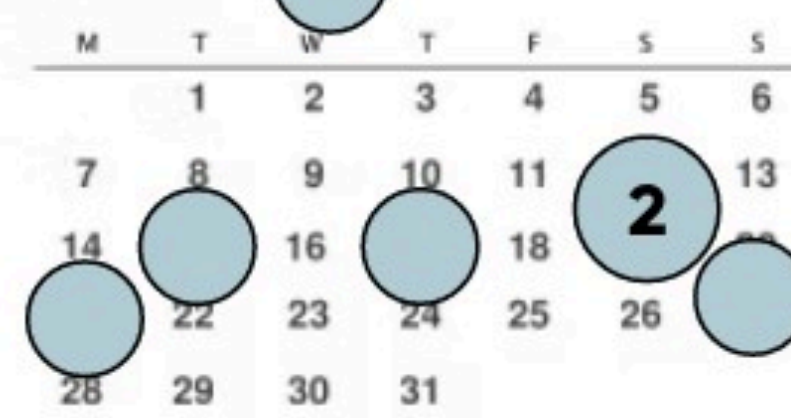
October



November

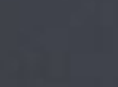
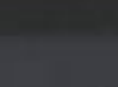
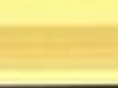
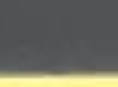
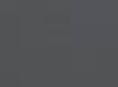
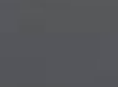
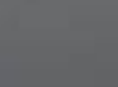


December



$D = 0 \parallel K \rightarrow C = 2^X \leftarrow \# \text{ bins}$

$\text{ball} = 1 \parallel K$



$B = \text{bins}$

$i = \# \text{ balls throw}$

$C_i = \{ \text{ball } i \text{ falls in a non-empty bin} \}$

$P[C_1] = 0$

$P[C_2] = \frac{1}{B}$

$P[C_3] = \frac{1}{B} P[C_3 \cap C_2] + P[C_3 \cap \bar{C}_2]$

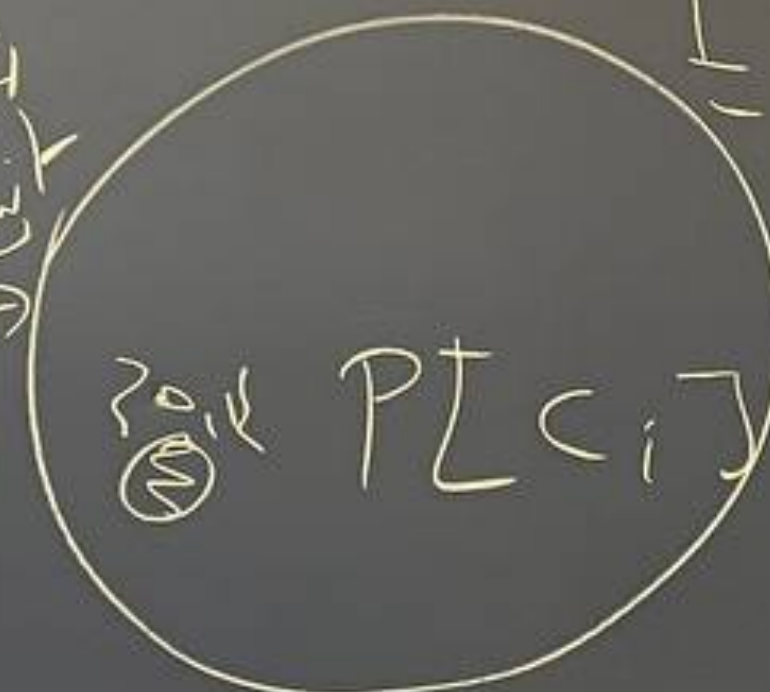
$\frac{1}{B} \cdot \frac{1}{B} + (1 - \frac{1}{B}) \cdot \frac{2}{B} = \frac{2}{B} - \frac{1}{B^2} \leq \frac{2}{B}$

$P[C_i] \leq \frac{i-1}{B} \quad \left\{ \begin{array}{l} i > B \\ i' = B - i < B \end{array} \right.$



$\left(\frac{1}{5}\right)^5$

0 1/2



reasoning on the blackboard (useful for HA1)